CSC 445 - Intro to Intelligent Robotics, Spring 2018

Feedback Control

Control

- Control theory deals with the control of continuously operating dynamical systems and understanding the behavior of dynamical systems.
- Example dynamical systems:
  - Robots
  - Epidemics
  - Biological systems
  - Stock markets

Basic Control Elements

- State of the system, a vector ξ
- Dynamic behavior of the system
- System of differential equations, $\dot{\xi} = f(\xi, u)$
- Control input that can affect the behavior, u
- Controller which takes some function of the desired state
- The output, y, that is a measurement of some aspect of the state

Example: Basic Control Elements for a Mobile Robot

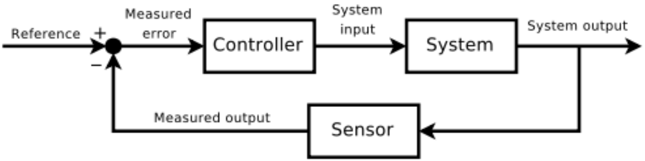
- State: $\xi = [x, y, \theta]^T$
- System of differential equations:

$$\dot{x} = v \cos\theta$$
$$\dot{y} = v \sin\theta$$
$$\dot{\theta} = \omega$$

- Control input: $u = [v, \omega]^T$

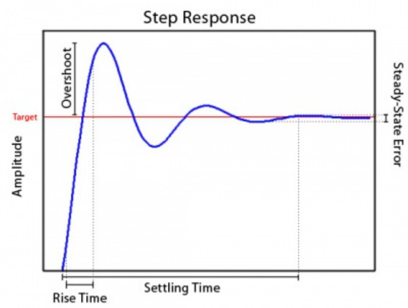
Feedback Control

- Feedback control deals with computing the control based on the output and the desired objective.
- Feedback controllers use the error (difference of output and the desired state) to compute a control input that drives the system error to zero.
- Feedback control loop:

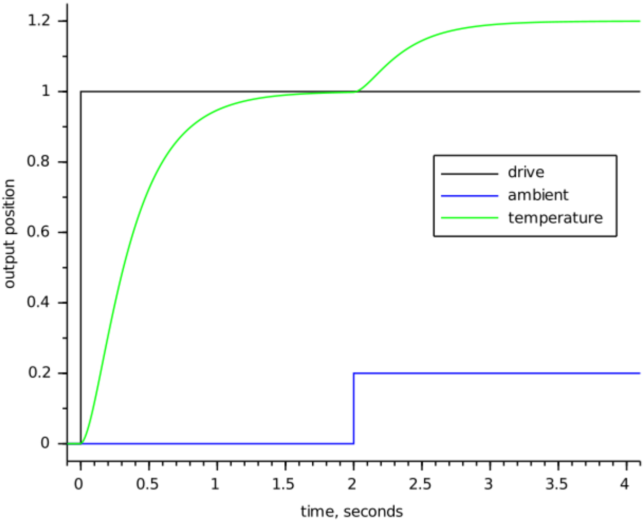


Characterizing Controller Performance

- Rise time: time to achieve desired value.
- Overshoot: largest magnitude in excess of desired value.
- Settling time:
- Steady-state error: error remaining after the controller input no longer affects plant output.



Example: Heater System Step Response



Reference: PID Without a PhD

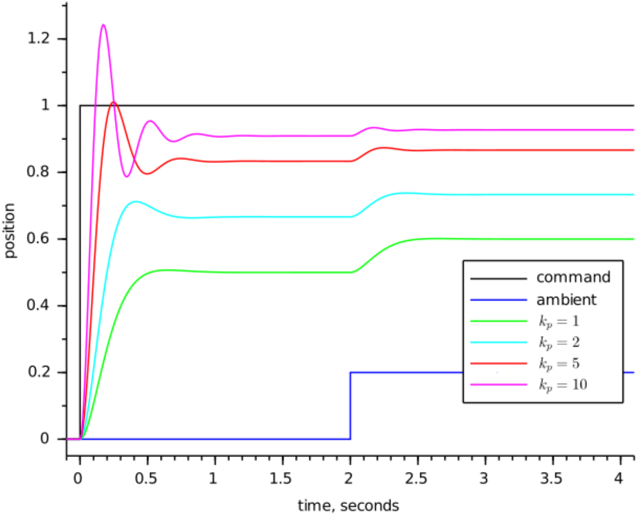
Proportional Controller

- The output is proportional to the input (error).

  $$u = k_p(\xi_d - \xi)$$

  where ξ_d is the desired state.
- The input is scaled by a proportional value, k_p, referred to as the controller gain.

Example: Heater P Controller



Reference: PID Without a PhD

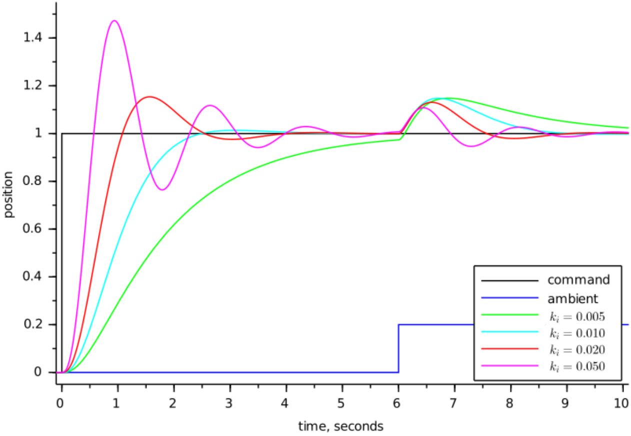
Integral (I) Controller

- The output is a function of an integral term:

  $$u = k_i \int_0^T (\xi_d - \xi) dt$$

  where k_i is the integral gain.
- The integral term basically "remembers" all that has happened since the beginning of time which allows it to cancel out any long term errors in the output.

Example: Heater I Controller



Reference: PID Without a PhD

Proportional Integral (PI) Controller

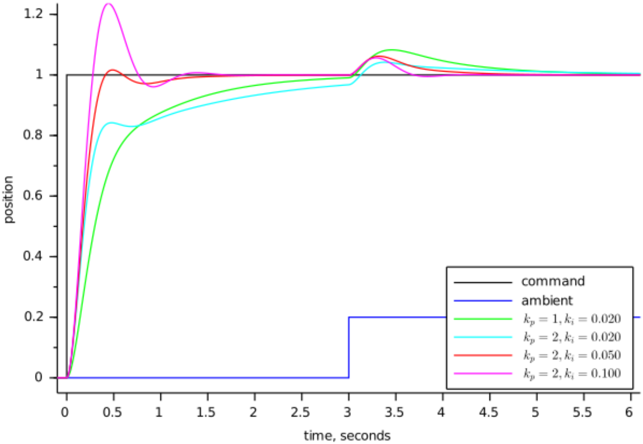
- The output is a function of a proportional term and an integral term:

$$u = k_p e + k_i \int_0^T e \, dt$$

  where $e = (\xi_d - \xi)$.

- The integral controller is not typically used by itself because some systems cannot be stabilized by an integral controller.

Example: Heater PI Controller



Reference: PID Without a PhD

Integral Controller Considerations

- The sampling time becomes important because the error is adding up over time.
- The integral term can cause significant overshoot as a result of integral windup – the integral term accumulates over time and dominates the control output.
- The integral term typically has a limit on its magnitude to mitigate integral windup.

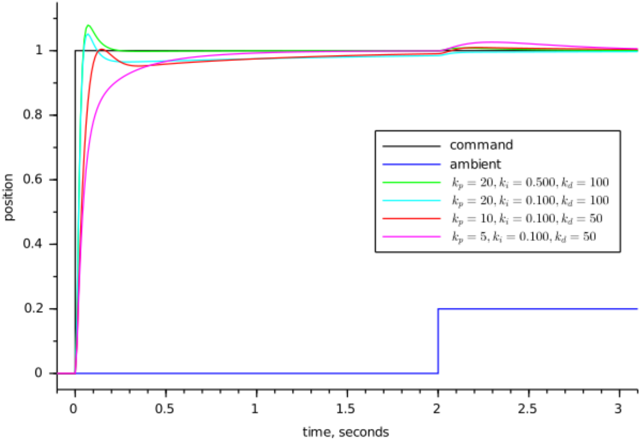
Proportional Integral Derivative (PID) Controller

- The output adds a derivative term to the PI controller

$$u = k_p e + k_i \int_0^T e \, dt + k_d \dot{e}$$

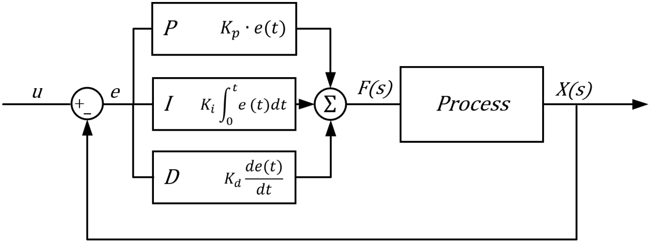
  where $e = (\xi_d - \xi)$.
- The derivative term is proportional to the rate of change of the error.
- Some systems cannot be stabilized without the derivative term.

Example: Heater PID Controller



Reference: PID Without a PhD

Proportional Integral Derivative Controller



Tuning PID Controller Gains

- The following table can be used a guideline for choosing controller gains.

Response	Rise Time	Overshoot	Settling Time	S-S Error
K_p	Decrease	Increase	Small Change	Decrease
K_i	Decrease	Increase	Increase	Eliminate
K_d	Small Change	Decrease	Decrease	Small Change